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Photon momentum and optical forces in cavities

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ABSTRACT

During the past century, the electromagnetic field momentum in material media has been under debate in the Abraham-Minkowski controversy as convincing arguments have been advanced in favor of both the Abraham and Minkowski forms of photon momentum. Here we study the photon momentum and optical forces in cavity structures in the cases of dynamical and steady-state fields. In the description of the single-photon transmission process, we use a field-kinetic one-photon theory. Our model suggests that in the medium photons couple with the induced atomic dipoles forming polariton quasiparticles with the Minkowski form momentum. The Abraham momentum can be associated to the electromagnetic field part of the coupled polariton state. The polariton with the Minkowski momentum is shown to obey the uniform center of mass of energy motion that has previously been interpreted to support only the Abraham momentum. When describing the steady-state nonequilibrium field distributions we use the recently developed quantized fluctuational electrodynamics (QFED) formalism. While allowing detailed studies of light propagation and quantum field fluctuations in interfering structures, our methods also provide practical tools for modeling optical energy transfer and the formation of thermal balance in nanodevices as well as studying electromagnetic forces in optomechanical devices.

Keywords: quantum optics, photon number, photon momentum, optical forces

1. INTRODUCTION

Investigations of radiation pressure and the momentum of light in dielectrics have frequently involved arguments about the correct form of the electromagnetic field momentum in material media.^{1–5} The Abraham and Minkowski forms for the single photon momentum are given by $\hbar k_0/n$ and $\hbar k_0 n$, which naturally depend on the vacuum wavenumber k_0 but also introduce contradicting and confusing dependencies on refractive index n . During the past century, powerful arguments have been advanced in favor of both momenta^{3,4} and various experimental setups measuring the forces due to light also seem to support both momenta.^{6–12}

In this work, we investigate the propagating field momentum and optical forces in cavity structures. The light pulses and single photon fields are described by using a field-kinetic theory that is based on the covariance principle,¹³ which states that the laws of physics are the same for all inertial observers. The steady-state field distributions are instead described by using the quantized fluctuational electrodynamics (QFED) formalism.^{14–16} The QFED approach for formulating the field operators is based on defining position-dependent photon ladder operators that obey canonical commutation relations.¹⁴ Our methods allow detailed studies of quantum field fluctuations in interfering structures, but also provide practical tools for modeling optical energy transfer and the formation of thermal balance in nanodevices as well as studying electromagnetic forces in optomechanical devices.

The manuscript is organized as follows: The field-kinetic model of photon propagation is presented in Sec. 2. Section 3 covers the principles of calculating electromagnetic forces in cavity structures by using the QFED model. Finally, conclusions are drawn in Sec. 4.

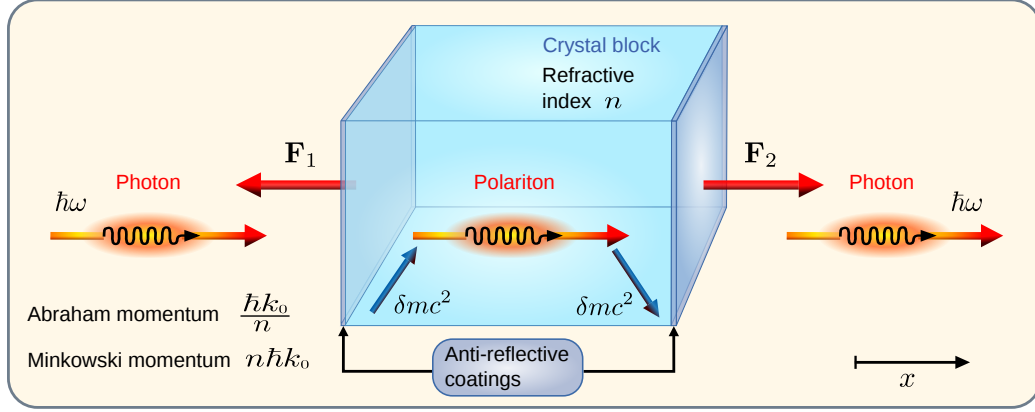


Figure 1. (Color online) Schematic illustration of a single photon transmission through a crystal block with refractive index n . The photon is first incident from vacuum. When entering the medium it couples to the induced electric dipoles in the medium forming a polariton quasiparticle. At the second end of the crystal the photon continues to propagate in vacuum. At the photon entrance and exit the medium block experiences forces \mathbf{F}_1 and \mathbf{F}_2 .

2. FIELD-KINETIC THEORY OF PHOTON PROPAGATION

In this section, we present theoretical observations regarding the single photon transmission through a crystal block with refractive index n illustrated in Fig. 1(a) by using a field-kinetic one-photon model. In our theory, we generalize the Feynman's description of light propagating in solids¹⁷ where the light quantum interacts with the induced atomic dipoles in the medium and forms a coupled state of light and matter that we call a polariton.

Here we use the concept of the polariton in a meaning that differs from its conventional use in the context of the phonon-polariton and the exciton-polariton quasiparticles. In these conventional cases the total energy of the polariton oscillates between two different eigenstates that represent physically different expressions of the total polariton energy. As discussed above, in our case the polariton is sooner a coupled state of electrons, ions, and the electromagnetic wave and it behaves like a Bloch state of electrons in solids. It propagates through the crystal without scattering preserving its energy and momentum and also conveys with itself a small but fixed amount of rest mass. It is also vital that the photon energy is far from the energy of the elementary electronic or ionic excitations of the medium so that absorption and spontaneous emission processes do not occur.

In the presentation of the field-kinetic one-photon model we first consider the energy and momentum conservation laws and the related covariance condition. The energy-momentum covariance only gives the relation between the energy and momentum, but does not uniquely determine the polariton momentum in the medium. Therefore, we also consider further physical conditions which can be used to determine the polariton momentum.

2.1 Energy and momentum conservation

We consider the total energy-momentum four-vectors of different parts of the composite system. These vectors consist of energy and momentum as $(E/c, p_x, p_y, p_z)$.¹³ The initial total four-momentum is written as a sum of the free photon four-momentum $P_0 = (\hbar k_0, \hbar k_0, 0, 0)$ and the four-momentum $P_M = (Mc, 0, 0, 0)$ for the medium block at rest as $P_{\text{tot}} = P_0 + P_M$.

In the field-kinetic description of the polariton, we assume that the induced dipoles of the medium carry a small but finite rest mass δm that will be determined from the conservation laws and the covariance conditions. The total polariton energy is then given by the sum of the initial electromagnetic energy $E_f = \hbar\omega$ and the rest energy $E_d = \delta mc^2$ corresponding to δm as $E = \hbar\omega + \delta mc^2$. In the medium, the total polariton energy E propagates with velocity $v = c/n$. The total momentum of the polariton denoted by p is a sum of the field and dipoles related contributions p_f and p_d and it will be uniquely determined in Sec. 2.2.

When the photon enters the medium, the photon couples with the atoms in the medium and the total energy and momentum of the system are shared by the propagating polariton $P_{\text{pol}} = (E/c, p, 0, 0)$ and the recoiling

Table 1. Polariton model energies and momenta calculated by using the general, Abraham, and Minkowski form polariton momenta p . Here $E = E_f + E_d$ and $p = p_f + p_d$ are the total energy and momentum of the polariton and the quantities with subscripts f and d are, respectively, related to the electromagnetic field and the induced dipoles.

	General	Abraham	Minkowski
E	npc	$\hbar\omega$	$n^2\hbar\omega$
E_f	$\hbar\omega$	$\hbar\omega$	$\hbar\omega$
E_d	$npc - \hbar\omega$	0	$(n^2 - 1)\hbar\omega$
p	p	$\hbar k_0/n$	$n\hbar k_0$
p_f	$\hbar k_0/n$	$\hbar k_0/n$	$\hbar k_0/n$
p_d	$p - \hbar k_0/n$	0	$(n - \frac{1}{n})\hbar k_0$

medium block $P_{\text{med}} = (M_r c, M_r V_r, 0, 0)$. Here $M_r = M - \delta m$ is the recoil mass of the medium block and V_r is the recoil velocity in the x -direction in Fig. 1. The total four-momentum must be conserved and thus we have $P_{\text{tot}} = P_{\text{pol}} + P_{\text{med}}$.

The unknown quantities of the model can be uniquely solved for a given total polariton momentum p by applying the energy and momentum conservation laws and the energy-momentum covariance condition $E^2/c^2 - p_x^2 - p_y^2 - p_z^2 = m_0^2 c^2$, where m_0 is the effective rest mass.¹³ The conservation of energy corresponds to the conservation of the first component of the four-momentum and it is written as

$$\hbar\omega + M c^2 = E + M_r c^2. \quad (1)$$

The momentum conservation instead corresponds to the conservation of the other components of the four-momentum and it is given for the nonzero second component by

$$\hbar k_0 = p + M_r V_r. \quad (2)$$

The principle of covariance also requires that the four-momenta obey the energy-momentum covariance condition. The covariance-condition-obeying energy $E = \gamma m_0 c^2$ and momentum $p = \gamma m_0 v$, where m_0 is the effective rest mass and $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor, obey $E = pc^2/v$. The covariance condition therefore directly relates the corresponding momenta $p_f = E_f v/c^2$ and $p_d = E_d v/c^2$ to the electromagnetic field and dipoles related energies $E_f = \hbar\omega$ and $E_d = \delta m c^2$.

By applying the conservation laws and the covariance condition, the induced dipoles related mass δm and the medium block recoil velocity V_r can be uniquely determined for a given total polariton momentum p as $\delta m = np/c - \hbar\omega/c^2$ and $V_r = (\hbar\omega - cp)/(M_r c)$. The energy and momentum contributions are presented in Table 1 for the general, Abraham, and Minkowski form polariton momenta p . It can be seen that, in the case of the Abraham momentum, the dipoles related quantities are zero, whereas, in the case of the Minkowski momentum, the total polariton quantities include dipoles related parts that make the total polariton momentum to be given by the Minkowski form.

Isolated systems like the photon plus the medium block in Fig. 1 are known to obey uniform motion described by a constant center of mass of energy velocity (CEV). According to our results, the CEV is written for the isolated system of a medium block and the photon before and after the photon has entered the medium as

$$V_{\text{CEV}} = \frac{\sum_i E_i v_i}{\sum_i E_i} = \frac{\hbar\omega c}{\hbar\omega + M c^2} = \frac{E v + M_r c^2 V_r}{E + M_r c^2}. \quad (3)$$

The equality of the numerators is nothing else than the conservation of momentum in Eq. (2) and the equality of the denominators corresponds to the energy conservation in Eq. (1). The above calculations are independent of the exact form of the polariton momentum p , which essentially shows that a covariant theory obeying the constant CEV motion can be formulated for both the Abraham and Minkowski momenta. This is essential as before only the Abraham momentum has been reasoned to obey the constant CEV motion.^{3,4} However, as described below, there exist further conditions that may uniquely define the polariton momentum.

2.2 Determination of the polariton momentum

As the above covariant theory does not uniquely define the exact form of the polariton momentum p , we next consider further physical conditions which can be used to directly determine the polariton momentum. One approach to determine the polariton momentum is given by the polariton Bloch state concept in which the polariton is considered to be a coupled Bloch state of light and matter. We follow the theory of the electronic structure of solids and suggest that the wavefunction of the polariton Bloch state, which can propagate through the medium without scattering or absorption, must be of the form $e^{i\mathbf{k}\cdot\mathbf{r}}u_{\mathbf{k}}(\tau)$, where \mathbf{r} is the position vector, \mathbf{k} is the wavevector of the polariton, and $u_{\mathbf{k}}(\tau)$ is the amplitude function which includes the microscopic structure of matter and its coupling with light through the generalized coordinate τ . The quantum mechanical momentum operator is given by $\hat{\mathbf{p}} = -i\hbar\nabla$, where ∇ is the vector differential operator. The momentum expectation value then obviously becomes $\langle\hat{p}\rangle = \hbar k$. Inside a medium, the wavelength of light decreases to $\lambda = \lambda_0/n$, where λ_0 is the wavelength in vacuum. Therefore, from the momentum expectation value, it directly follows that $\langle\hat{p}\rangle = \hbar k = 2\pi\hbar/\lambda = n2\pi\hbar/\lambda_0 = n\hbar k_0$ and thus the Minkowski momentum is a natural consequence of the polariton Bloch state concept. This will be discussed further in the forthcoming manuscript where we also use the Lorentz force law and the Maxwell's equations to show that this simplified model is indeed fully consistent with the semiclassical continuum picture and the energy-momentum tensors of light and matter.

2.3 Physical consequences

One of the most evident physical consequences of our analysis is that energy corresponding to the rest mass δm is effectively attached to the photon when the photon propagates inside the medium as a polariton quasiparticle. Note, however, that the nonzero rest mass is a property of the polariton not the photon. As we obtained the Minkowski momentum for the polariton, we have $\delta m = (n^2 - 1)\hbar\omega/c^2 > 0$, which essentially means that the polariton transfers the mass δm from the first to the second interface of the block and the medium is left in a nonequilibrium state which later on returns to equilibrium through relaxation processes. Since the photon energy is conserved in the transmission process, the energy of this nonequilibrium state is very close to the energy of the initial state and the relaxation processes are practically elastic.

As an example of the Minkowski form polariton momentum for $\hbar\omega = 1$ eV and $n = 2$, we have $\delta mc^2 = 3$ eV. If the mass density of the medium is 1000 kg/m^3 , this mass transfer effectively corresponds to a displacement of a medium cube with side length $2 \times 10^{-13} \text{ m}$, which corresponds to a small fraction of an atom. This displacement is divided into a region of many atoms and one can well expect that the energy required to produce such a small displacement of atoms along the path of the photon is meaningless in a dispersionless medium and the photon does not lose energy in the transmission process. One can also respectively speculate that in dispersive media the atomic displacements may be inelastic when the photon loses energy which leads to dispersion.

Our results suggest that the experimental measurements that directly measure the polariton momentum or the force due to a light beam inside a medium⁶⁻⁹ must give the Minkowski momentum for the polariton. The Abraham momentum seems to be only supported by indirect measurements¹⁰⁻¹² and theoretical arguments that do not take the polariton associated rest mass and the related mass transfer and its relaxation into account.

The uniquely defined polariton model quantities corresponding to the Minkowski form polariton presented in the last column in Table 1 are illustrated in Fig. 2. Figure 2(a) shows the total polariton energy and its contributions associated to the electromagnetic field and the induced electric dipoles in the medium as a function of the refractive index. The total polariton energy in Fig. 2(a) increases as a function of the refractive index due to the increasing polariton associated rest mass. The increased rest mass is also related to the reduction of the propagation velocity of light in the medium. The energy contribution associated to the electromagnetic field part of the coupled polariton state remains constant $\hbar\omega$.

The polariton momentum is presented as a function of the refractive index in Fig. 2(b). The Minkowski form total polariton momentum increases linearly with the increasing refractive index. Its electromagnetic field contribution given by the Abraham momentum instead decreases with the increasing refractive index. The difference of the Minkowski and Abraham momenta is carried by the induced electric dipoles in the medium.

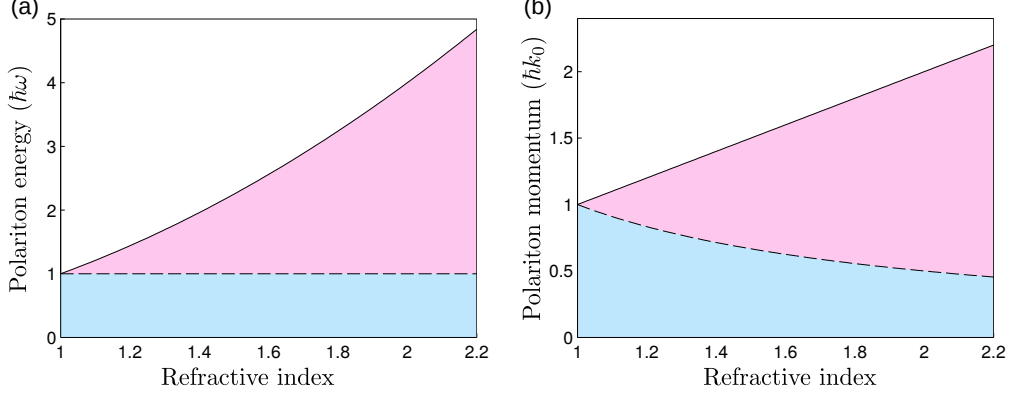


Figure 2. (Color online) (a) Polariton energy and (b) polariton momentum as a function of the refractive index. The solid lines denote the total polariton energy $E = E_f + E_d$ and momentum $p = p_f + p_d$ and the dashed lines denote the electromagnetic field associated parts $E_f = \hbar\omega$ and $p_f = \hbar k_0/n$ of the corresponding quantities. The differences of the solid and dashed lines correspond to the energy and momentum parts $E_d = \delta mc^2 = (n^2 - 1)\hbar\omega$ and $p_d = (n - \frac{1}{n})\hbar k_0$ carried by the induced electric dipoles in the medium.

3. QUANTIZED FLUCTUATIONAL ELECTRODYNAMICS

To provide additional insight on the forces present at the interfaces in Fig. 1, we briefly review the methods to calculate the forces \mathbf{F}_1 and \mathbf{F}_2 of the figure by using the QFED formalism.^{14–16,18} Fully equivalent steady-state forces can also be obtained directly from the corresponding Maxwell's stress tensor. Since, in the QFED formalism, we study time-independent steady-state fields, we do not face the problem of defining the photon momentum and the Abraham-Minkowski controversy. In the QFED, the forces are calculated by using the operator form of the classical Maxwell's stress tensor. It follows that the x -component of the spectral force density expectation value is given by¹⁵

$$\langle \hat{\mathcal{F}}_x(x, t) \rangle_\omega = -\frac{\hbar\omega}{2} \left(\frac{\partial}{\partial x} \rho(x, \omega) \right) - \hbar\omega \left(\frac{\partial}{\partial x} \rho(x, \omega) \right) \langle \hat{n}(x, \omega) \rangle - \hbar\omega \rho(x, \omega) \frac{\partial}{\partial x} \langle \hat{n}(x, \omega) \rangle, \quad (4)$$

where $\rho(x, \omega)$ is the electromagnetic local density of states (LDOS) and $\langle \hat{n}(x, \omega) \rangle$ is the position-dependent photon number expectation value. The first term in Eq. (4) corresponds to the familiar zero-point Casimir force (ZCF),^{19,20} the second term is known as the thermal Casimir force (TCF),^{21–23} and the last term arising from the changes in the total photon number is called a nonequilibrium Casimir force (NCF)¹⁵ since it disappears at thermal equilibrium when the derivative of the photon number is zero. The net force on an area S of a solid object extending from x_1 to x_2 can then be obtained by integrating the force density in Eq. (4) as $\langle \hat{F}(t) \rangle_\omega = S \int_{x_1}^{x_2} \langle \hat{\mathcal{F}}_x(x, t) \rangle_\omega dx$.¹⁵ Equivalently, the net force can be also obtained by using the concept of electromagnetic pressure. The electromagnetic pressure along the x direction is given by¹⁵

$$\langle \hat{\mathcal{P}}(x, t) \rangle_\omega = \hbar\omega \rho(x, \omega) \left(\langle \hat{n}(x, \omega) \rangle + \frac{1}{2} \right). \quad (5)$$

Therefore, the net force on an object extending from x_1 to x_2 can be obtained as $\langle \hat{F}(t) \rangle_\omega = S[\langle \hat{\mathcal{P}}(x_1, t) \rangle_\omega - \langle \hat{\mathcal{P}}(x_2, t) \rangle_\omega]$.¹⁵

The photon numbers propagating to the left and right in different parts of the cavity geometry corresponding to the medium block in Fig. 1 are illustrated in Fig. 3. As there are no losses, the left and right propagating and total field photon numbers are piecewise continuous and only depend on the cavity geometry and the input fields $\langle \hat{n}_{1+} \rangle$ and $\langle \hat{n}_{3-} \rangle$ incident from the left and right. The refractive indices of the three media are given by

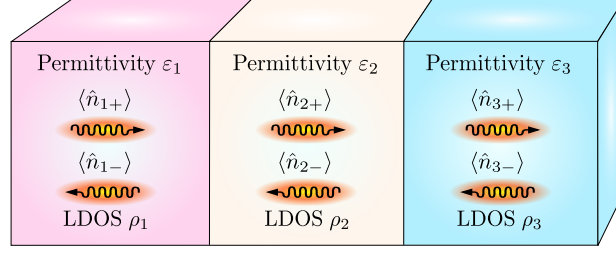


Figure 3. (Color online) Optical cavity structure consisting of three homogeneous layers. The left and right propagating field photon-number expectation values and LDOSs are presented in each layer.

$\sqrt{\varepsilon_1}$, $\sqrt{\varepsilon_2}$, and $\sqrt{\varepsilon_3}$. In different regions of the geometry, the propagating photon numbers are written as¹⁶

$$\begin{aligned}
 \langle \hat{n}_{1-} \rangle &= |\mathcal{R}_1|^2 \langle \hat{n}_{1+} \rangle + \sqrt{\varepsilon_1/\varepsilon_3} |\mathcal{T}'_1 \mathcal{T}'_2|^2 \langle \hat{n}_{3-} \rangle, \\
 \langle \hat{n}_{2+} \rangle &= \frac{\sqrt{\varepsilon_2/\varepsilon_1} |\mathcal{T}_1|^2 \langle \hat{n}_{1+} \rangle + \sqrt{\varepsilon_2/\varepsilon_3} |\mathcal{T}'_2 \mathcal{R}'_1|^2 \langle \hat{n}_{3-} \rangle}{\text{Re}[1 + 2\mathcal{R}'_1 \mathcal{R}_2 \nu_2 e^{2ik_2 d_2}]}, \\
 \langle \hat{n}_{2-} \rangle &= \frac{\sqrt{\varepsilon_2/\varepsilon_1} |\mathcal{T}_1 \mathcal{R}_2|^2 \langle \hat{n}_{1+} \rangle + \sqrt{\varepsilon_2/\varepsilon_3} |\mathcal{T}'_2|^2 \langle \hat{n}_{3-} \rangle}{\text{Re}[1 + 2\mathcal{R}'_1 \mathcal{R}_2 \nu_2 e^{2ik_2 d_2}]}, \\
 \langle \hat{n}_{3+} \rangle &= \sqrt{\varepsilon_3/\varepsilon_1} |\mathcal{T}_1 \mathcal{T}_2|^2 \langle \hat{n}_{1+} \rangle + |\mathcal{R}'_2|^2 \langle \hat{n}_{3-} \rangle,
 \end{aligned} \tag{6}$$

where d_2 is the width of the cavity, k_2 is the wavenumber inside the cavity, $\nu_2 = 1/(1 + r_1 r_2 e^{2ik_2 d_2})$, $\mathcal{R}_1 = (r_1 + r_2 e^{2ik_2 d_2})\nu_2$, $\mathcal{R}_2 = r_2$, $\mathcal{T}_1 = t_1 \nu_2$, $\mathcal{T}_2 = t_2$, $\mathcal{R}'_1 = r'_1$, $\mathcal{R}'_2 = (r'_2 + r'_1 e^{2ik_2 d_2})\nu_2$, $\mathcal{T}'_1 = t'_1$, and $\mathcal{T}'_2 = t'_2 \nu_2$ with the conventional single interface Fresnel reflection and transmission coefficients for left incidence r_i and t_i , $i \in \{1, 2\}$, and right incidence r'_i and t'_i , $i \in \{1, 2\}$. In contrast to the electric and magnetic field values where resonance effects can substantially increase the magnitude of the field inside a resonator, the photon-number values inside the cavity and at the outputs in Eq. (6) are always between the input field photon numbers. This essentially ensures that in global thermal equilibrium all the photon numbers are equal and no photon-number accumulation can occur inside the cavity at the equilibrium state.

The total force on the medium block due to a light beam incident from vacuum can be calculated as a difference of electromagnetic pressures on both sides of the cavity multiplied with the area S . The LDOSs on different sides of the cavity are equal and, therefore, by using the spectral electromagnetic pressure in Eq. (5) the total spectral force due to a light beam becomes

$$\langle \hat{F} \rangle_\omega = \frac{\langle \hat{n}_1 \rangle - \langle \hat{n}_3 \rangle}{\langle \hat{n}_{1+} \rangle} \langle \hat{F}_0 \rangle_\omega, \tag{7}$$

where $\langle \hat{F}_0 \rangle_\omega$ is the spectral force in the case of a perfect reflector in vacuum. When applying the propagating photon numbers in Eq. (6) and the identity $|\mathcal{R}_1|^2 + |\mathcal{T}_1 \mathcal{T}_2|^2 = 1$, the force in Eq. (7) becomes $\langle \hat{F} \rangle_\omega = |\mathcal{R}_1|^2 \langle \hat{F}_0 \rangle_\omega$. The force is thus naturally proportional to the total power reflection coefficient of the structure $|\mathcal{R}_1|^2$. If the medium block would be coated with anti-reflective coatings, the total force would be zero meaning that the interface forces on the first and the second interface cancel each other in accordance with the field-kinetic one-photon model in Sec. 2. The spectral interface forces are, in this case, explicitly given by $\langle \hat{F}_1 \rangle_\omega = (1 - n) \langle \hat{F}_0 \rangle_\omega$ and $\langle \hat{F}_2 \rangle_\omega = (n - 1) \langle \hat{F}_0 \rangle_\omega$, where $n = \sqrt{\varepsilon_2}$ is the refractive index of the medium block.

4. CONCLUSIONS

In conclusion, our analysis suggests that when a photon enters the crystal its energy and momentum will be shared by the crystal and the propagating light wave or the polariton. As the ratio of energy and momentum of the polariton is different from that of light in vacuum, the light wave can not be covariantly described by a pure photon state that has no rest mass. The covariance can be restored by assuming that the polariton propagating in a crystal is a coupled state of a photon and the induced dipoles in the medium with a small but

finite polariton rest mass. In contrast to the previous interpretations that only the Abraham momentum would obey the constant CEV motion of an isolated body, we have shown that the constant CEV motion can also be obeyed by the polariton with the Minkowski momentum. We have also used the QFED formalism to study the steady-state interface forces in the corresponding medium block geometry.

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